

Turunan Implisit

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11:34 AM

Review

$$\text{Aturan Turunan : } (u \pm v)' = u' \pm v'$$

$$(u \cdot v)' = u'v + u \cdot v'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

Aturan Rantai:

$$y = f(u(x)) \Rightarrow \frac{dy}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

$$z = f(y(x)) \Rightarrow \frac{dz}{dx} = \frac{df}{dy} \cdot \frac{dy}{dx}$$

$$z = x^2 \cdot \sin x \Rightarrow \frac{dz}{dx} = 2x \cdot \sin x + x^2 \cos x$$

$$z = y^2 \cdot \sin y \Rightarrow \frac{dz}{dy} = 2y \cdot \sin y + y^2 \cos y$$

$$z = \sin u \Rightarrow \frac{dz}{dx} = \cos u \cdot \frac{du}{dx}$$

$$z = x^2 + y^2 \Rightarrow \frac{dz}{dx} = 2x + 2y \cdot \frac{dy}{dx}$$

$$z = y^2 \Rightarrow \frac{dz}{dx} = 2y \cdot \frac{dy}{dx}$$

$$z = x^2 + y^2 \Rightarrow \frac{dz}{dx} = 2x + 2y \cdot \frac{dy}{dx}$$

$$z = x^2 + y^2 \Rightarrow \frac{dz}{dy} = 2x \cdot \frac{dx}{dy} + 2y$$

$$z = y^2 \Rightarrow \frac{dz}{dx} = 2y \cdot \frac{dy}{dx}$$

$$z = x^2 \cdot y \Rightarrow \frac{dz}{dx} = 2x \cdot y + x^2 \cdot 1 \cdot \frac{dy}{dx}$$

$$z = x^2 y^3 \Rightarrow \frac{dz}{dy} = 2x \cdot \frac{dx}{dy} \cdot y^3 + x^2 \cdot 3y^2$$

$$z = x^{10} y^5 \Rightarrow \frac{dz}{dx} = \frac{10x^9 y^5 - x^{10} \cdot 5y^4 \cdot \frac{dy}{dx}}{(y^5)^2}$$

$$z = x^2 \sin y \Rightarrow \frac{dz}{dx} = 2x \cdot \sin y + x^2 \cdot \cos y \cdot \frac{dy}{dx}$$

Tentukan $\frac{dy}{dx}$ with soal $\frac{?}{?}$ berikut ini

$$1) x^2 + y^2 = 25 \Rightarrow \frac{dy}{dx} = ?$$

Jawab:

$$\frac{d}{dx} (x^2 + y^2) = \frac{d}{dx} (25)$$

$$\Rightarrow 2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow 2y \cdot \frac{dy}{dx} = -2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{2y} //$$

$$2) y = x^2 + y^3$$

$$\Rightarrow \frac{d}{dx} (y) = \frac{d}{dx} (x^2) + \frac{d}{dx} (y^3)$$

$$\Rightarrow 1 \cdot \frac{dy}{dx} = 2x + 3y^2 \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 2x$$

$$\Rightarrow (1 - 3y^2) \frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{2x}{(1 - 3y^2)}$$

$$3) \quad x^2 + 2xy + y^2 = 9$$

$$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(2xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(9)$$

$$\Rightarrow 2x + \left(2y + 2x \cdot 1 \cdot \frac{dy}{dx} \right) + 2y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow 2x + 2y + 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow (2x + 2y) \frac{dy}{dx} = -2x - 2y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y} = -1$$

EXAMPLE 2

- (a) Find y' if $x^3 + y^3 = 6xy$. $\rightarrow \frac{dy}{dx}$ $\rightarrow m = \frac{dy}{dx} \Big|_{\substack{x=3 \\ y=3}}$
- (b) Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point (3, 3).
- (c) At what points on the curve is the tangent line horizontal?

EXAMPLE 3 Find y' if $\sin(x + y) = y^2 \cos x$.

$y = m(x - x_1) + y_1$
 $m = 0$

5-20 III Find dy/dx by implicit differentiation.

5. $x^2 + y^2 = 1$

6. $x^2 - y^2 = 1$

7. $x^3 + x^2y + 4y^2 = 6$

8. $x^2 - 2xy + y^3 = c$

9. $x^2y + xy^2 = 3x$

10. $y^5 + x^2y^3 = 1 + ye^{x^2}$

11. $x^2y^2 + x \sin y = 4$

12. $1 + x = \sin(xy^2)$

13. $4 \cos x \sin y = 1$

14. $y \sin(x^2) = x \sin(y^2)$

15. $e^{x^2y} = x + y$

16. $\sqrt{x + y} = 1 + x^2y^2$

17. $\sqrt{xy} = 1 + x^2y$

18. $\tan(x - y) = \frac{y}{1 + x^2}$

19. $xy = \cot(xy)$

20. $\sin x + \cos y = \sin x \cos y$

21. $x = y^n$