

# Turunan Fungsi dan Aturan Rantai

Monday, August 30, 2010

11:37 AM

Turunan fungsi  $y = f(x)$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

(Turunan y TERHADAP x, biasanya had variabelnya)

Notasi Turunan:  $y', f', \frac{dy}{dx}, \frac{d}{dx}(y), \frac{d}{dx}(f), D_x$

1)  $f(x) = x^2$

$$f(x+\Delta x) = x^2 + 2x\Delta x + \Delta x^2$$

2)  $f(x) = x^3$

$$f(x+\Delta x) = x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3$$

				1		
			1	1		
		1	2	1		
		1	3	3	1	
		1	4	6	4	1
	1	5	10	10	5	1

4)  $f(x) = (x+\Delta x)^n = x^n + n \cdot x^{n-1} \cdot \Delta x + \dots$  Rumus Binomial

$$(x+\Delta x)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} \cdot \Delta x^k$$

Tugas ① Cari Rumus Binomial beserta penjelasannya.  
(biasanya ada di buku Discrete Mathematics)

①  $f(x) = x^n$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^n - x^n}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\left( x^n + n \cdot x^{n-1} \cdot \Delta x + \binom{n}{2} x^{n-2} \cdot \Delta x^2 + \dots + \Delta x^n \right) - x^n}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{n \cdot x^{n-1} \cdot \Delta x + \binom{n}{2} x^{n-2} \cdot \Delta x^2 + \dots + \Delta x^n}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} n x^{n-1} + \binom{n}{2} x^{n-2} \cdot \Delta x + \dots + \Delta x^{n-1}$$

$$= n \cdot x^{n-1} + 0 + 0 + \dots + 0 = n x^{n-1}$$

$$\therefore \frac{d}{dx}(x^n) = n \cdot x^{n-1}, \text{ atli n bulat}$$

Utuk n yg tidak bulat ternyata menghasilkan rumus yg sama

yaitu  $\frac{d}{dx}(x^a) = a \cdot x^{a-1}$ , Bukti: Mgg Dpn !!

# Aturan Turunan

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2  $f(x) = u(x) + v(x)$ , Tentukan  $f' = (u+v)' = u' + v'$   
 ↓  
 Tunjukkan !!!

$$f(x+\Delta x) = u(x+\Delta x) + v(x+\Delta x)$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{u(x+\Delta x) + v(x+\Delta x) - (u(x) + v(x))}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{(u(x+\Delta x) - u(x)) + (v(x+\Delta x) - v(x))}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{u(x+\Delta x) - u(x)}{\Delta x} + \lim_{\Delta x \rightarrow 0} \frac{v(x+\Delta x) - v(x)}{\Delta x}$$

$$= u'(x) + v'(x)$$

∴  $\frac{d}{dx}(u+v) = u' + v'$

3)  $f(x) = \frac{1}{v(x)} \Rightarrow f(x+\Delta x) = \frac{1}{v(x+\Delta x)} \Rightarrow f = \frac{1}{v}$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{v(x+\Delta x)} - \frac{1}{v(x)}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{v(x) - v(x+\Delta x)}{v(x+\Delta x) \cdot v(x)}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{\Delta x}{v(x) - v(x+\Delta x)}}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{-v'(x) + v'(x)}{v(x+\Delta x) \cdot v(x) \cdot \Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-(v(x+\Delta x) - v(x))}{\Delta x} \cdot \frac{1}{v(x+\Delta x) \cdot v(x)}$$

$$= -v'(x) \cdot \frac{1}{v(x+\Delta x) \cdot v(x)} = \frac{-v'(x)}{(v(x))^2}$$

$$\Rightarrow f' = \frac{-v'}{v^2}$$

# Aturan Turunan dan Aturan Rantai

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$$4) f(x) = u(x) \cdot v(x) \quad \Bigg\| \quad (u \cdot v)' = u'v + u \cdot v'$$

$$5) f(x) = \frac{u(x)}{v(x)} \quad \Bigg\| \quad \left(\frac{u}{v}\right)' = \frac{u'v - u \cdot v'}{v^2}$$

$$6) f(x) = \sin x \Rightarrow f'(x) = \cos x \quad (\text{lihat Diklat})$$

$$7) f(x) = \cos x \Rightarrow f'(x) = -\sin x$$

$$8) f(x) = \tan x = \frac{\sin x}{\cos x} \Rightarrow f'(x) = \sec^2 x$$

$$9) f(x) = \sec x = \frac{1}{\cos x} \Rightarrow f'(x) = \sec x \cdot \tan x$$

Tugas 1: Buktikan Aturan no 4, 5, 7, 8, 9

dan menggunakan definisi turunan:

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

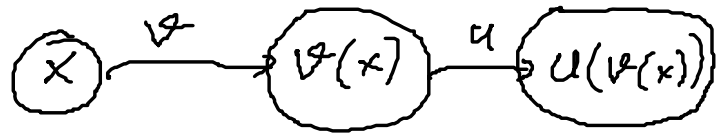
(Silahkan buktikan kembali referensi dari diklat)

# Aturan Rantai

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Komposisi Fungsi:



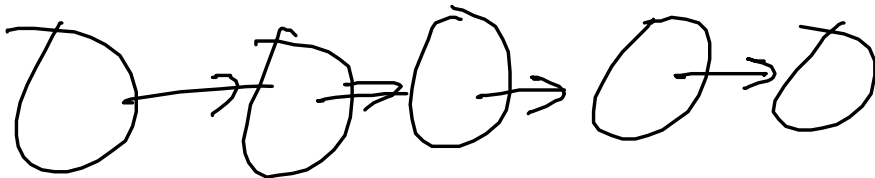
$$f(x) = u(v(x))$$

$$x \xrightarrow{x^2+3} (x^2+3) \xrightarrow{v^4} (x^2+3)^4$$

$$x \rightarrow (x+3x^2) \rightarrow (x+3x^2)^7$$

$$x \rightarrow x^2+3x \rightarrow \sin(x^2+3x)$$

$$x \rightarrow x^2+2 \rightarrow \sin(x^2+2) \rightarrow (\sin(x^2+2))^7$$



$$f(x) = u(v(x)) \Rightarrow \frac{d}{dx}(f(x)) = \frac{du}{dv} \cdot \frac{dv}{dx}$$

Bukti Formal: Lihat hal 14 di kitab paku Nag.

Bukti Non Formal (sebagai pengingat saja).

$$f(x) = u(v(x)).$$

$$\frac{\Delta u}{\Delta x} = \frac{\Delta u}{\Delta v} \cdot \frac{\Delta v}{\Delta x}$$

} ambil saja  
 $\Delta x \rightarrow 0$ , dan  
 $\Delta v \rightarrow 0$

$$\frac{du}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$

{ Turunan u thd v  
dikalikan turunan v  
berhadap x }

# Contoh penggunaan Aturan rantai

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- Tentukan  $\frac{dy}{dx}$  untuk soal-soal berikut ini.

1)  $f(x) = (x^2 + 2x)^5 \Rightarrow v(x) = x^2 + 2x$ , dan  
 $u(v) = v^5$ , diperoleh:

$$\frac{dv}{dx} = 2x + 2, \quad \frac{du}{dv} = 5v^4 = 5(x^2 + 2x)^4$$

$$\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx} = 5(x^2 + 2x)^4 \cdot (2x + 2)$$

$$f(x) = (x^2 + 2x)^5 = 5(x^2 + 2x)^4 \cdot (2x + 2)$$

Turun
inside
Turun  
outside
Tetap
inside

2)  $y = (x^3 + 2x)^7 \Rightarrow y' = 7(x^3 + 2x)^6 \cdot (3x^2 + 2)$

3)  $y = \sin^3(x^2) \Rightarrow y' = 3(\sin^2(x^2)) \cdot \cos(x^2) \cdot (2x)$

4)  $y = \cos^2 x \Rightarrow y' = 2\cos(x) \cdot (-\sin x) \cdot (1)$

5)  $y = \sqrt{x^3 + 5} = (x^3 + 5)^{1/2} \Rightarrow y' = \frac{1}{2}(x^3 + 5)^{-1/2} \cdot (3x^2)$

6)  $y = (x^3 + 2)^2 + x^2 \Rightarrow y' = 2(x^3 + 2) \cdot (3x^2) + 2x$

7)  $y = \frac{1}{5x} = (5x)^{-1} \Rightarrow y' = -1 \cdot (5x)^{-2} \cdot (5) = \frac{-5}{25x^2}$

8)  $y = \frac{1}{\cos x} = \sec x \Rightarrow y' = \sec x \cdot \tan x$

# Latihan Soal

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$$27. y = x^5 + 5x^4 - 10x^2 + 6$$

$$28. y = 3x^{1/2} - x^{3/2} + 2x^{-1/2}$$

$$29. y = \frac{1}{2x^2} + \frac{4}{\sqrt{x}} = \frac{1}{2}x^{-2} + 4x^{-1/2}$$

$$30. y = \sqrt{2x} + 2\sqrt{x}$$

$$31. f(t) = \frac{2}{\sqrt{t}} + \frac{6}{\sqrt[3]{t}}$$

$$32. y = (1 - 5x)^6$$

$$33. f(x) = (3x - x^3 + 1)^4$$

$$34. y = (3 + 4x - x^2)^{1/2}$$

$$35. \theta = \frac{3r+2}{2r+3}$$

$$36. y = \left(\frac{x}{1+x}\right)^5$$

$$37. y = 2x^2\sqrt{2-x}$$

$$38. f(x) = x\sqrt{3-2x^2}$$

$$39. y = (x-1)\sqrt{x^2-2x+2}$$

$$40. z = \frac{w}{\sqrt{1-4w^2}}$$

$$41. y = \sqrt{1+\sqrt{x}}$$

$$42. f(x) = \sqrt{\frac{x-1}{x+1}}$$

$$43. y = (x^2+3)^4(2x^3-5)^3$$

$$44. s = \frac{t^2+2}{3-t^2}$$

$$45. y = \left(\frac{x^2-1}{2x^3+1}\right)^4$$

Tugas 1c: no 30,31,37,38,39,40,42,45